4. The Evolution of Commercial Arithmetic

> Let no man venture into large business while he is ignorant of the method of regulating books; never let him imagine that any degree of natural abilities will enable him to supply this deficiency, or preserve multiplicity of affairs from extricable confusion. (Samuel Johnson, *A Dictionary of the English Language*, 1755, v.1, Preface).

Commercial Arithmetic and Commercial Education

The rudimentary beginnings of modern financial economics can be traced to the early commercial arithmetics. While scholastic doctrine dictated how financial economics *ought* to be done, the early commercial arithmetics are reflections of actual practice. Some of these commercial arithmetics were influenced by the scholastic Church school educational process, with the commercial arithmetic being part of a larger study that also featured the theoretical aspects of arithmetic, algebra and geometry. Other commercial arithmetics were wholly practical, designed only for use in reckoning schools and commercial practice. From the time of the Renaissance, the important commercial arithmetics were primarily algorisms, though the older abacus methods were still popular in some locales until well into the 16th century.

The association of business schools with universities is a relatively modern development. Traditionally, universities and Church schools used a curriculum that had a scholarly emphasis on the humanities. Use-oriented subjects, such as engineering, applied mathematics and business studies, were of secondary importance. Starting in the 16th century, this neglect of practical subjects at the expense of humanist pursuits was increasingly questioned by scholars such as Petrus Ramus at the University of Paris. Though there was considerable resistance from the academic hierarchy, there was a gradual trend toward acceptance of use-oriented subjects, as evidenced by the establishment in 1600 of the Dutch Mathematical School, associated with the University of Leiden. This trend was further amplified during the 17th

century by the dramatic intellectual advances in astronomy, mathematics, physics, engineering and other use-oriented subjects.

Despite the post-17th century trend toward giving increased academic importance to use-oriented subjects, adequate business education was still difficult, if not impossible, to obtain within the universities and Church school system. Those desiring a detailed commercial education, such as merchant apprentices, were obliged to attend other institutions, such as the reckoning schools or their successors. Reckoning schools, where reckoning masters accepted students for private tuition or conducted formal group classes, appeared in Italy prior to the 14th century and spread to northern Europe along the trade routes. Students came, typically, from merchant families, usually following a grammar school education. The reckoning school functioned as an essential component in the training of merchant apprentices. The reckoning master also acted as a consultant on various types of often complicated problems involving commercial calculations.

Commercial arithmetic was an essential component of the reckoning school curriculum. This meant that commercial arithmetic was typically taught by reckoning masters operating outside the universities. The texts on commercial arithmetic which reckoning masters published for use in their classrooms were almost always exclusively practical, early versions of the modern business school textbooks. The reckoning school as an institution evolved over time. This evolution occurred to suit the profound changes in business practices that took place during the 16th and 17th centuries. For example, the increasing importance of bookkeeping and accounting led to increasing emphasis on that subject in the reckoning school curriculum.

The reckoning school, as an institution, was also adapted to suit the particular characteristics of the countries into which it was transplanted. In England and Scotland, the reckoning school evolved into the commercial academies and 'writing schools' that were the backbone of commercial education during the 17th and 18th centuries. By the late 17th century, instruction in handwriting had risen to a level of importance equal to that of instruction in commercial arithmetic and accounting. The English writing master had come to replace the reckoning master as the purveyor of commercial education. The traditional reckoning master, a specialist in the 'arte of numbers', was now more of a consulting specialist. Compared with the 15th and 16th centuries, by the 18th century the gap between reckoning school education and the knowledge of the consulting specialists was

substantial; only the most capable of the writing masters, such as James Dodson, was able to make the transition to being a consulting specialist.

The Treviso Arithmetic

While Italy was the centre for European commercial education, by the time of the printing of the *Treviso* there were numerous important commercial centres outside of Italy where reckoning masters, or algorists, were in increasing demand. Lyons was one such centre, where a flourishing spice market contributed to the success of the Lyons fairs following the royal protection provided to the fairs by Louis XI in 1464. It was in Lyons where Nicholas Chuquet, a master algorist, worked and by 1484 completed a series of manuscripts referred to as the *Triparty*. While the *Triparty* is arguably the most significant theoretical contribution among the early commercial arithmetics, a number of other less important commercial arithmetics, in manuscript form, also had emerged in the various commercial centres in the time between Fibonacci's *Liber abaci* and the appearance of the *Treviso Arithmetic*.

The *Treviso Arithmetic* (1478) is not a particularly memorable book from the standpoint of high theory. The primary relevance of the book is that it was the first printed book on commercial arithmetic and, more generally, on mathematics. As such, the book almost certainly provides a snapshot of the teachings that an unnamed *maestri d'abbaco* gave to the students of his reckoning school. The book is untitled and draws its name from being an arithmetic published in Treviso, a town in the Venetian republic, some 26 kilometres northwest of Venice. Treviso was a town of some economic importance, being located on the main trade route about one day's journey on the route linking Venice with northern and central European centres such as Vienna and the German cities. Economic activity in Treviso was sufficiently robust that, as early as 1372, the town was capable of supporting a *maestri d'abbaco* (Swetz 1987, ch. 1).

The significance of the *Treviso* in the early history of financial economics lies more with what the book represents than with what it contains. The *Treviso* was the first printed mathematics book, though there were other potential candidates for this honour with considerably greater scholastic interest such as the works of Euclid. The book was written by some anonymous reckoning master intended as a self-study practica for those involved in commercial trades. It is written in the Venetian dialect, not in Latin which was the language of scholarly

instruction in the medieval and Renaissance universities.[1] As such, the *Treviso* represents the importance and advanced development that Venetian commercial education and the subject of commercial arithmetic had achieved by the latter part of the fifteenth century.

The contents of the *Treviso Arithmetic* were translated to English by the noted mathematics historian David Eugene Smith in 1907 with the text of the translation given, in full, by Swetz (1987). There is no title page, table of contents or index. The original folio has 62 double pages and the text is not broken into chapters. After a brief discussion of numbers, the *Treviso* proceeds to discuss addition, subtraction, multiplication and division, with illustrations often involving monetary units, which complicates the discussion but is a necessary feature of commercial arithmetic. This consumes approximately half of the text. Starting on the thirtieth folio page, the rule of three is examined and treated with numerous examples. This consumes the next 28 pages of text. After a brief examination of computi calculations, the *Treviso* concludes with a listing of a number of practical rules useful in mercantile calculations, for example, 'Note that to make pounds of ounces, divide the ounces by 12'.

The *Treviso Arithmetic* did not turn out to be a popular book. The book appears to have had only one edition and copies of the book today are extremely rare. Much more successful than the *Treviso Arithmetic* was the *Libro De Abacho* by Pietro Borgo (or Borghi) which first appeared in 1484 and went into no less than sixteen editions, with the last edition appearing in 1577. D. Smith (1926) calls the *Libro De Abacho* the 'First Great Commercial Arithmetic' and recognizes that a 'book so valued as to go through at least sixteen editions in a century must have had and evidently did have a great influence on the work of later generations' (p.49). The original and later editions of the book were published in Venice and the book was written in the Venetian dialect. Little is known of the author other than what is on the title page, his name and the appellation, 'da veniesia', Peitro Borgo of Venice.

The *Libro De Abacho* is an unabashed practical commercial arithmetic, stating explicitly that the types of theoretical numbers that were of interest to the Greeks and Boethius were to be discarded and only those of use in commercial applications were to be examined. The text follows the general approach of Fibonacci's *Liber abaci* by starting with a discussion on the reading and writing of numbers and then proceeding to cover multiplication, division, addition and then subtraction, in that order. In contrast, the *Treviso Arithmetic* is

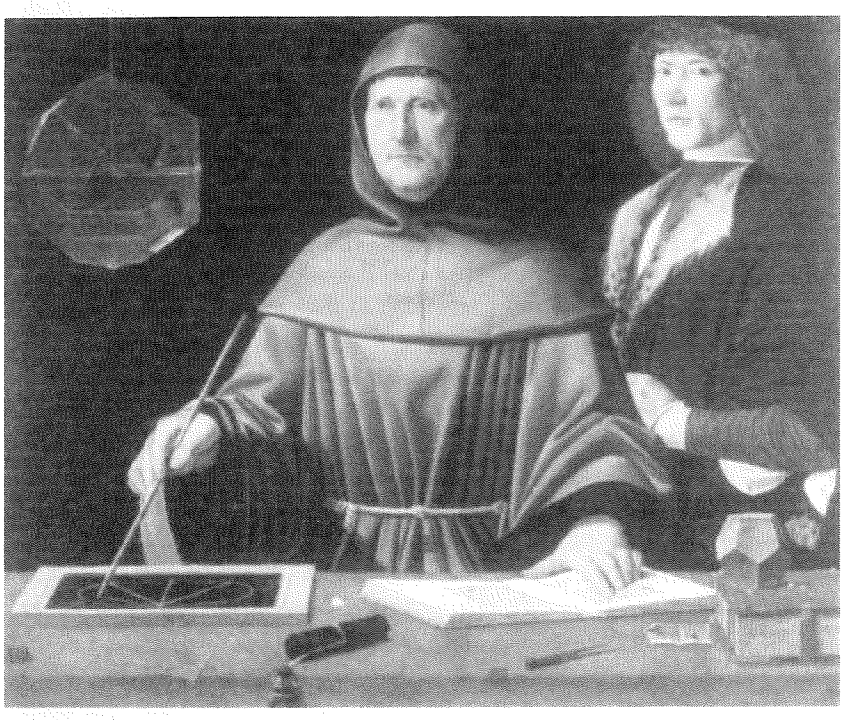
strikingly modern in covering the arithmetic operations by proceeding from addition to subtraction to multiplication and then division. Both books proceed to deal with fractions and the Rule of Three, essential for calculations involving currency exchange and barter. Both books end with a substantial number of practical problems, with worked solutions provided.

Pacioli and the *Summa*

Even though the *Libro De Abacho* may have been the most important *practica* in the period from the late 15th to the end of the 16th century, it is little more than a curiosity to modern researchers. The accessibility of printing to commercial education marked by the publication of the *Treviso Arithmetic* led to a surge of published mathematics works printed in Italy. From 1480-1490, at least 63 mathematical works were printed in Italy alone (D. Smith 1958, v.1, p.41), rising to 214 by 1500 and 1,527 by 1600 (D. Smith 1958, v.1, p.249). Of all the commercial and mathematical texts produced during this period, one text still receives considerable attention from modern researchers, the *Summa de Arithmetica, Geometria, Proportioni et Proportionalita* (1494) by Fra Luca Paciolo (1445-1523?), also known as Luca Pacioli (Paccioli), Lucas Patiolus, Luca Paciuolo, Luca Paciuolo de Borgo san Sepulcro (D. Smith 1970, p. 54) and Fra Luca di Borgo, the name that Pacioli took upon entering the Franciscan order (Figure 4.1).

The most important claim to priority associated with Pacioli and the *Summa* is that the *Summa* was the first printed treatise on double entry accounting. The bookkeeping part of the *Summa*, which is composed of 36 chapters and is entitled 'Of Reckonings and Writings' (*De computis et scripturis*), had an influence on accounting practice that was profound. In the 100 years following original publication, the bookkeeping part of the *Summa* had been translated into at least five languages, the 1543 English translation by Hugh Oldcastle, a London reckoning master, is the oldest known treatise on accounting in English. Until approximately the 19th century, the 'method of Venice' dominated accounting education and practice. When modern accounting historians refer to the *Summa* it is with words like: 'The treatise by Pacioli has been the foundation upon which practically all subsequent writings concerning double entry bookkeeping have been predicated' (Green 1930, p.41).

Figure 4.1 Fra Luca Pacioli



In addition to the section on bookkeeping, the *Summa* has also been recognized for a minor contribution to the history of statistics. There is one example from the *Summa*, concerned with dividing the stakes from a game of chance that terminates prior to completion, that captures the essence of the 'problem of points', a topic of fundamental importance in the history of statistics, for example, David (1962, p.37): 'A and B are playing a fair game of *balla*. They agree to continue until one has won six rounds. The game actually stops when A has won five and B three. How should the stakes be divided?' Unfortunately, Pacioli's solution provided to the stated 'problem of points' is incorrect (5:3 instead of the correct solution of 7:1), which substantively undermines his importance in that subject. It is unlikely that Pacioli recognized the probabilistic implications of the problem.

What is Double-Entry Bookkeeping?

Double-entry bookkeeping is based on the duality of every business transaction. Due to disagreements over the origination of double-entry, it is historically useful to establish precise requirements for what constitutes double-entry. One essential requirement is that all transactions are recorded twice, once on the credit or asset side and once on the debit or liability + equity side. This requirement is the essence of double entry which leads to the basic balance sheet equation of the double-entry system:

$$\text{Assets} = \text{Liabilities} + \text{Equity}$$

Instead of Equity, the terms Capital or Net Worth are also used. For example, when a partnership purchases an asset, say a ship, then the funds which are used to purchase the asset will be the counterbalance; the double-entry item will be reflected by incurring a liability such as a loan, an infusion of equity reflected in an increased investment in the firm by the ownership or an another asset which was sold to raise the needed cash.

The duality of the balance sheet has an immediate counterpart in ongoing transactions. Ongoing revenues are associated with increasing assets such as cash balances, paying down of liabilities or decreasing the ownership claim by distributing the revenues to the ownership. It follows that another fundamental element in the double-entry system is that there will be a balance in the summation of the dual items. This balancing of the books is as important a component of double-entry as the dual recording of transactions. While the presentation of the accounts in a bilateral form is useful, de Roover (1956) reports that accountants in 14th century Tuscany achieved the requirements of double-entry before the adoption of the bilateral, or Venetian, tabular presentation.

Unlike the *Treviso Arithmetic* and the *Libro De Abacho* which had purely commercial education in mind, the *Summa* is written in the manner of *Libre Abaci* and other scholastic tracts. However, unlike the *Libre Abaci* which was written in Latin, the *Summa* is written in the Venetian dialect. The text of the *Summa* covers five topics (Chatfield 1974, ch. 4): algebra and arithmetic; the use of these subjects in business; bookkeeping; money and exchange; and, pure and applied

geometry. Though the book was published in Venice in 1494, it appears as though much of the text was written about seven years earlier when Fra Luca was at Perugia. The primary purpose of the *Summa* was not to give instruction in bookkeeping, the subject for which it has claimed the most fame in modern times. Rather, the objective was to summarize the mathematical knowledge available at the time. While the book does contain some previously unpublished work by Pacioli, it is generally maintained that the *Summa* is 'a remarkable compilation with almost no originality' (D. Smith 1958, v.1, p.252).

Biographies of individuals living in Renaissance Italy are often difficult to obtain. The modern practice of writing laudatory biographies or self-promoting autobiographies was uncommon. Life was usually absorbed in the service of the Church and state. For individuals such as Pietro Borgo, writing to further the interests of *mei signori Merchandanti Venetiani* was a sufficient reward. Though sketchy in parts with some disagreement on specific points in the available sources (for example, Green 1930; Taylor 1942; Chatfield 1974; David 1962), much of Pacioli's life is quite traceable. Little is known of the early life of Pacioli, other than that he was born about 1445 at Borgo Sancti Sepulchri in the Province of Arezzo in Tuscany. He apparently received a Church school education from the Franciscan friars and may have received some instruction and academic influence from a fellow townsman, Pietro Franceschi (1410?-1492), an artist who also produced a respectable work on geometry (D. Smith 1958, v.1, p.251).

Around the age of twenty, Pacioli travelled to Venice and became a tutor in mathematics to the three sons of the wealthy merchant, Ser Rompiasi. During the six years Pacioli spent in this position, he engaged in the study of mathematics and, just prior to his leaving for Rome in 1471, he completed writing an algebra that was never published. Because of the need to provide instruction to the sons of merchants, it is likely that he became well acquainted with commercial arithmetic and bookkeeping during this period. Upon arriving in Rome, Pacioli entered the Minorite order of the Franciscans, taking the name Fra Luca di Borgo. Whether Pacioli undertook becoming a Franciscan for purely religious reasons is unknown. However, in Renaissance Italy, becoming a Franciscan was a desirable means of both obtaining university education and securing employment as a teacher in Church schools and universities (Chatfield 1974, p.44).

The next period of Pacioli's life is taken up with travel, teaching and education. The actual route of his travels is unknown, though there

were extensive excursions in Italy and possibly in the Orient. He held a teaching position in Perugia for a number of years starting from 1475 and another position in Zara in Dalmatia in 1481. By this time, Pacioli had gained recognition as a superior lecturer in mathematics and in 1482 he received the honour of being called to Rome to serve as a mathematics lecturer. Having travelled extensively and earned the equivalent of a doctorate, Pacioli returned to Perugia in 1486-1487 and commenced writing the bulk of the text for the *Summa*. Over the next eight years, Pacioli taught mathematics in various centres such as Naples, Pisa, Venice and finally Urbino as well as giving public lectures in Rome and Venice. It was in Urbino that the text of the *Summa* was completed, as evidenced by the dedication to the Duke of Urbino.

The last third of Pacioli's life that followed the publication of the *Summa* involved more travel, teaching and writing. In the period from 1496-1499, Pacioli was a professor of mathematics at Milan where he held a position in the court of Duke Ludovico Sforza il Moro (Lewis the Moor). It was in Milan that Pacioli met Leonardo da Vinci, who was said to spend evenings 'telling stories to ladies of the court and joining in their pastimes and games' (David 1962, p.36). Pacioli and da Vinci become acquainted to the point where da Vinci produced figures for a text that Pacioli was writing on geometry, *De diuina proportione*. D. Smith (1958, v.1, p.254) claims the Pacioli borrowed much of the material for this book from Franceschi. After holding positions in Pisa, Rome, Florence, and Venice, in 1510 he returned to his birthplace to head the Monastery of San Sepulcro. Details of his death are uncertain but Pacioli was apparently alive in 1523 when the second edition of the *Summa* was published.

Plagiarism

Aside from the reverence given to the *Summa* in accounting, modern opinion on the contribution made by Pacioli with the *Summa* is decidedly negative. Numerous sources express an overt indignation about the degree of plagiarism contained in the text. David (1962, p.37) states that 'Fra Luca was not a great mathematician and his importance lies in the fact that he summarized the mathematical learning that was then common'. David observes that the *Liber abaci* is incorporated almost in its entirety. D. Smith (1958, v.1, p.252) states that Pacioli in the *Summa* 'borrowed freely from other sources, often without giving the slightest credit', recognizing material that had been

borrowed from Euclid, Ptolemy, Fibonacci and Boethius among others. D. Smith also observes that Pacioli was 'a careless writer, so much so that Cardano has a chapter devoted to errors in his book' and that his summary of Euclid was 'poor'. Boyer (1968, p.307) states that the *Summa* 'was more influential than original' and that Pacioli 'borrowed freely' from the technical commercial arithmetics available at the time.

There is a sharp contrast between the opinions of the historians of mathematics and accounting. For example, Chatfield (1974, p.45) states that 'the *Summa* was unsurpassed for 100 years even by imitators who wrote with Pacioli's text in hand ... Pacioli was most remarkable for his possession of qualities which are common enough in themselves but are hardly ever found together in the same mind'. Green (1930, p.40) is almost apologetic in stating: 'There is no evidence that Pacioli laid claim to having originated the method of double entry bookkeeping. He specifically stated that he followed the method which was then utilized in Venice.' Green (p.41) is unambiguous in stating that: 'The treatise by Pacioli has been the foundation upon which practically all subsequent writings concerning double entry bookkeeping have been predicated.'

Unlike the uniformly negative opinions of Pacioli derived from mathematicians, there is not complete agreement among accountants. For example, de Roover (1956, p.178) states that 'in the fifteenth century, the practice of the counting-house was far ahead of the rather simple system described in the early treatises, including the first and most celebrated one, by ... Luca Pacioli'. Elsewhere, de Roover claims that much of the bookkeeping material in the *Summa* was 'more or less reworked' from a manuscript of another author circulating in the reckoning schools of Venice around that time. While recognizing the importance of the *Summa* in the development of accounting, Brown (1905, p.120) does not agree about the importance of the *Summa* in spreading knowledge of accounting to the masses. Rather, 'Pacioli uses no examples, and without the assistance of these the art can hardly be acquired by a novice. Pacioli addresses himself to experts.'

All of this discussion raises a number of issues. One concern is the problem of attributing the origin of ideas to specific individuals. The relevance of this issue goes beyond the simple problem of fairness, giving credit where credit is due, which is a valid reason in and of itself. Resolving how the progress of knowledge takes place relies on identifying where and how ideas are generated. A tremendous amount of intellectual energy has been given to this issue through the years, with the Newton-Leibnitz disputation over the origination of the

calculus being but one well-known example. In this regard, the history of financial economics is dramatically different from the histories of mathematics and statistics. In the early history of financial economics, it is often not possible to identify precisely where a specific idea originated.

Pacioli, in particular, raises another fundamental issue concerning the role of summarization in the process of generating knowledge. Careful consideration of this issue leads to evaluation of the 'David principle' concept where ideas generated by junior scholars are attributed to more senior scholars. Another variation of the David principle has well-positioned scholars receiving credit for ideas that are actually reworked versions of ideas generated by less well-positioned scholars. In an extreme case, these issues speak to the problem of identifying plagiarism. The intellectual history of the period under study is replete with instances where one or more of these considerations is raised. However, resolution of the intellectual problems raised is beyond the pale of this study. All that is attempted here is a chronicle of the evolution of the ideas, with an honest attempt being made to recognize any originator that can be identified.

The task of preparing a chronology and identifying contributors to early financial economics is not easy. In the early part of the period, records are often scarce. This is particularly so for personal records, due at least partially to the social practice of dedicating one's life to Church and State. Records for individuals operating in the scholastic realm are somewhat more accessible than for those involved purely in commercial practice. This paucity of personal information is compounded by the intellectual practice of borrowing liberally from previous sources without giving appropriate credit, with the *Summa* being a classic example. This practice appeared to be widespread and may have been understood by contemporaries, though as time passed attempts to practice 'acceptable plagiarism' generated numerous controversies.

At what point the practice of 'acceptable plagiarism' became unacceptable is unclear. In financial economics, a number of fierce disputes appear in the late 17th and early 18th centuries, the dispute between Abraham de Moivre and Thomas Simpson (1710-1761) over the valuation of life annuities being perhaps the most well known. In mathematics the disputes over origination start much earlier. The debate between Nicolo Tartaglia (1506-1557) and Lodovico Ferrari (1522-1560), sparked by the blatant plagiarism of Ferrari's mentor Girolamo Cardano (1501-1576), over the solution to the cubic and

quartic equations is well known, for example, David (1962, ch. 5), but is interesting enough to bear repeating.

Around 1525 Nicolo Fontana, better known as Tartaglia, devised a method for solving a wide range of cubic equations, at a time when only a restricted number of cubic equations could be solved. Under considerable pestering by the prolific Cardano, in 1537 Tartaglia partially revealed his secret solution procedure, swearing Cardano to secrecy. When one of Cardano's students, Ludovico Ferrari, developed a solution for a quartic equation in 1540 using the procedure, the result was one of the bitterest scholastic disputes of the 16th century. The debate was sparked by the 1545 publication of Ferrari's result in Cardano's *Artis Magnae Liber* where Cardano passes the quartic result off as his own. Ferrari's relationship with Cardano is an example of the how the David principle operated at that time.

Bibliographic and autobibliographic information improves progressively from the 16th century onward. This is due to a range of factors, including the relentless expansion of print media. In turn, as education and research evolved beyond scholastic confines, potential gains of social status and material well being were associated with making academic contributions. Instead of writing for Church and countryman with no desire for recognition, contributions were increasingly being assessed and individual recognition given. For example, the Royal Society was founded in 1660 with membership being determined by recognized works. In conjunction, the practice of 'acceptable plagiarism' also evolved. Sometimes heated disputes arose after the origination of specific ideas, with the Newton-Leibnitz controversy being the most prominent. 'Acceptable plagiarism' still survived in the pamphlet and policy literature, with Josiah Child being an example (Letwin 1964).

Chuquet and the *Triparty*

The general scope of the early commercial arithmetics has been sketched. The tradition of the commercial arithmetics was distinct from the classical mathematics associated with the Church schools and universities. Renaissance commercial needs required vernacular texts oriented to explaining the calculations required to make increasingly complicated commercial transactions. In general, European commercial arithmetics were similar in form to the *Treviso Arithmetic* and the *Libro Di Abacho*. The texts were written in the vernacular and designed for use by merchants or in reckoning schools; effectively commercial

arithmetics functioned as early business school textbooks. These books did not contribute significantly to the theoretical development of financial economics but, rather, reflected the business practices of the time.

Related to the purely commercial arithmetics were texts that incorporated a commercial arithmetic into a broader mathematical framework. These books were closer to the scholastic tradition that considered arithmetic as one of the arts. A commercial arithmetic was included as part of a text that also dealt with subjects such as geometry, number theory in the tradition of Boethius and algebra. These texts, while less accessible to a merchant audience, do reflect the progress in commercial arithmetic. Certain texts, such as Nicolas Chuquet's *Triparty en la Science des Nombres* (1484), contain significant contributions to other areas of mathematics, such as algebra in the case of the *Triparty*. By implication, such texts are also relevant to financial economics because the quality of the author signals whether the commercial arithmetic contained in these works is likely to be the best available at that time.

Compared to Pacioli, almost nothing is known about Nicolas Chuquet (1440?-1500). The date and place of his birth is unknown, though it is possible he was born in Paris. Information from tax registers indicates that he started working in Lyons, France in 1480 as an *escripvain*, literally a copyist or writing master. Lyons at this time was an important French commercial centre. The Lyons fairs, specializing in the spice trade, had been bolstered by royal protection granted by Louis XI in 1464. In addition to the economic activity of the fairs, Lyons was also a banking centre and was developing a printing industry. In this environment, an *escripvain* performed the role of copying various commercial and legal documents. In this capacity, it is likely that Chuquet came into frequent contact with the sizable Italian community of Lyons, members of which were actively involved in economic affairs.

In addition to supporting a functionary such as an *escripvain*, Lyons was also a centre where a reckoning master would be in demand. By 1485, the tax registers of Lyons list Chuquet as an algorist, indicating that his mathematical skill had been recognized and he had progressed to where he was working as a reckoning master. The quality of mathematics contained in the *Triparty* clearly indicate that mathematical skill and knowledge of commercial arithmetic had spread well beyond the confines of Italy by this time. The sources that influenced Chuquet can only be guessed. It is possible that he visited Italy, and there is

strong evidence that he was familiar with important Italian mathematical texts of the time.

The immediate impact of Nicolas Chuquet's *Triparty* on the development of commercial arithmetic is difficult to assess. Writing outside Italy, Chuquet's contributions had little apparent influence on the study of commercial arithmetic in Italy which was the centre of reckoning school education at that time. Modern mathematical opinion maintains that the bulk of the commercial arithmetic contained in the *Triparty* was 'fairly conventional' (Flegg et al. 1985, p.24), but this is not surprising, given that commercial arithmetic reflected business practices that changed little in the 16th century due to the 'slowness with which the customs of commerce changed' in that period (D. Smith 1926). In the history of mathematics, the *Triparty* is recognized for the important insights the text had in algebra, with the proviso that the text on algebra was so advanced for its time that it was neglected. The same comment cannot be attributed to the section of the *Triparty* concerned with commercial arithmetic.

For the history of financial economics, the *Triparty* is important for its explicit recognition and treatment of compound interest and for the use of the 'rule of first terms' in dealing with interest calculations (Benoit 1988). Because the manuscript was not published until Aristide Marre resurrected it in 1880 (Flegg et al. 1985), it is unlikely that the *Triparty* had any direct influence. However, as a reckoning master, Chuquet certainly passed on his insights to his students and it is through this avenue that his impact was made.

Though there are few references to Chuquet's work by 16th century mathematicians, substantial portions of his *Triparty* manuscript were incorporated wholesale into the 1520 publication, *Larismethique nouellement composee*, of Estienne de la Roche, probably a former student of Chuquet. As a contribution to algebra, Marre considers the *Larismethique* to be 'vastly inferior' to the *Triparty*, though much of Chuquet's contribution to algebra can be ascertained from this text. The *Larismethique* did have an impact as a commercial arithmetic. According to D. Smith (1958, v.1, p.313): 'Perhaps no other French arithmetic of the 16th century gives a better view of the methods of computation and commercial applications of the subject.'

Even though France was not a leading commercial nation in the 16th century, Lyons was one location within France that was of continental importance as a commercial and banking centre. To quote de Roover (1949, p.108), around the middle of the 16th century 'the principle banking centres in Europe were Antwerp, Lyons, Genoa and the fairs

of Castille'. The importance of Lyons as a banking centre almost certainly contributed to the emergence of the Lyons school of commercial arithmetic. In addition to Chuquet and de la Roche, the Lyons school also included Jacques Peletier (1517-1582) and Jean Trenchant (1525-?). *L'Arithmetique* (1558,1566) published in Lyons by Trenchant is noteworthy in providing tables needed for compound interest calculations (Lewin 1970).

As the 16th century progressed, social attitudes toward payment of interest slowly evolved and this was reflected in the commercial arithmetics. In a detailed examination of fourteen French commercial arithmetics written during the 16th century, all published in French, Davis (1960, pp.22-4) finds ten explain the method of computing compound interest with four of those criticizing the practice but still providing an explanation in order to be in accord with regular business practice. The earliest of the texts examined by Davis (1960) was written by two monks, one Spanish and one French, and published in 1515. This text gives an uncritical explanation of compound interest: 'Here follows another rule for lending money, by which the profit or interest earns as well as the principal.'

Progress of Commercial Arithmetic

During the 16th century, numerous published contributions to commercial arithmetic appeared throughout Europe. These works were usually written in the native tongue, though works in Latin were still produced. With some refinements, the content of the vernacular texts typically corresponded with what was contained in Borghi while the Latin texts continued to feature a commercial arithmetic as part of a wider study of arithmetic. The availability of commercial arithmetics in specific languages roughly approximated the extent of commercial development. As in the 15th century, important contributions were usually from mathematicians though not always of the highest stature.

In Germany, where commerce was active, Jacob Köbel (1470-1533) and Adam Riese (1489-1559) published the first editions of their influential German language works in 1514 and 1518. Köbel's most influential arithmetic, *Rechenbiechlin*, went into 22 editions during the 16th century while Riese's book has been characterized as 'one of the most popular textbooks of the century', consistent with the view that Riese was the 'greatest of all the Rechenmeisters' of the century (D. Smith 1958, v.1, p.337). German commercial arithmetic of this period is concerned with progressing from the older abacus methods, based on

Figure 4.2 A woodcut from the title page of Adam Riese's Rechenbach *(1529)*

Note: The theme of the woodcut is concerned with the controversy between algorists and abacists, which was quite heated at the time that Riese was writing.
Source: This image was adapted from Swetz (1987).

counters and Roman numerals, to written calculation using pen and Hindu-Arabic numerals. Riese is recognized as being the most influential writer in this movement. As a mathematician, he also made contributions to algebra (Figure 4.2).

Compared to Germany, England was less commercially developed at this time and this is reflected in the development of commercial arithmetic. The first substantive English contribution to commercial arithmetic in the century appeared in Latin in 1522 by Cuthbert Tonstall, a non-mathematician remembered more for his diplomatic and

ecclesiastical contributions. This work lacked originality but was useful in summarizing the approach used in the Italian commercial arithmetics. Tonstall undoubtedly acquired the relevant background during his time spent at Padua while acquiring his doctor of laws. In any event, Tonstall's work is more of historical interest. Being a Latin text, the book lacked popularity as a commercial arithmetic.

Though there were a number of previous arithmetics published in English, the first commercial arithmetic with any influence is Robert Recorde's (1510-1558) *The Ground of Artes* (1542) which covers both abacus and Hindu-Arabic calculation methods. This book went into 18 editions by the end of the century, with a further 11 editions in the 17th century. Unlike Tonstall, Recorde was a mathematician, arguably 'just about the only mathematician of any stature in England throughout the (16th) century' (Boyer 1968, p.317). In addition to his popular commercial arithmetic, Recorde is recognized by historians of mathematics for making contributions to both astronomy and geometry.

The only other popular English commercial arithmetic to appear during the 16th century was *The Well Spring of Sciences* (1562) by Humphery Baker. Much as with Borghi, little is known of Baker other than his being from London and that he died after 1587. Judging from the contents of his book, it is likely that Baker was an English reckoning master reproducing material used in his daily work and, possibly, his reckoning school classes. The book borrows liberally from Recorde and follows the approach of using a combination of abacus and Hindu-Arabic approaches. One clue as to the motivation for the book is contained in the author's preface which indicates that the book 'was written to meet the criticism of continental scholars on the backward state of (commercial arithmetic) in England' (D. Smith 1958, v.1, p.321).

The Low Countries, comprising what is now Belgium and Holland, have a diverse collection of 16th century commercial arithmetics. Consistent with an advanced stage of commercial development, there were a large number of vernacular commercial arithmetics. As much of the merchant populous was multilingual, a range of French commercial arithmetics was available. In addition to arithmetics emanating from France, some Dutch and Flemish authors elected to publish in French. The list of authors includes Valentin Menher, a reckoning master, whose three or four arithmetics, published in the second half of the century, 'occupied the same position in the Netherlands that Borghi's did in Italy and Recorde's did in England' (D. Smith 1958, v.1, p.342).

In addition to works in French, the Low Countries also produced arithmetics in Latin, including an important and popular work by Gemma Frisius (1508-1555), the 'most influential of the various Dutch mathematicians of (the) century', also known as Gemma Regnier. Frisius is second only to Simon Stevin in importance among 16th century Low Country mathematicians. Unlike England, due to the wider use of Church school education, the merchant class in the Low Countries had a firmer basis in Latin, allowing greater accessibility to the *Arithmeticae Practicae Methodus Facilus* (1540). As expected from a mathematician writing in Latin, both theoretical and commercial topics are examined in the book. The popularity of the arithmetic is evidenced by the 59 editions of the book published in the 16th century, with several more in the 17th century.

As for books in the Dutch vernacular, the first printed book appears in 1508, initially printed by Thomas Vander Noot with an enlarged version appearing in 1510, printed by Willem Vorsterman. Part of the 1508 arithmetic appears in a 1527 printing by Jan Severz. The English translation of the 1508 title, 'The way of learning to calculate according to the true art of Algorisms in whole and broken numbers', suggests that the contents are a commercial arithmetic. The 1510 version is enlarged to include a section on the abacus arithmetic topic of reckoning with counters. Only two other Dutch arithmetics are known to have been printed in the first half of the 16th century, an untitled 1532 printing by Christianus van Varenbraken, also a commercial arithmetic, and a 1537 arithmetic by Gielis Vanden Hoecke, 'On the Noble Art of Arithmetic'. Little is known about any of the authors (Kool 1988).

The second half of 16th century finds a dramatic change in the publication of vernacular Dutch commercial arithmetics, with numerous books being introduced. As a group, these works have historical importance in being a reflection of the growth in the social depth and importance of the merchant class in Holland. However, none of the writers of these arithmetics can make individual claims of historically important contributions to commercial arithmetic. The main impact of these contributions was to disseminate commercial arithmetic to those working primarily in Dutch.

As it turns out, during this period the most important contribution to commercial arithmetic originating from the Low Countries was contained in a section of an arithmetic published originally in Flemish and republished in the same year in French. The historical importance of *La Pratique d'Arithmetique* (1585) as a commercial arithmetic is due to at least as much to the reputation of its author, Simon Stevin, as to

the contents of the text. However, the commercial arithmetic is only one part of a larger text that is devoted primarily to theoretical arithmetic. The contribution of the other parts of the arithmetic, in particular to the development of decimal fractions, is more noteworthy. Of particular importance to financial economics, the topic of interest calculations is given a long chapter, 'with excellent tables for facilitating its calculation' (Murray 1930, p.215).

Simon Stevin and Leiden

Simon Stevin (1548-1620) was born in Bruges and, at a young age, was placed in a merchant's office in Antwerp. Stevin probably gained a useful exposure to commercial practice during the brief period he was in this position, following which he travelled extensively to countries such as Prussia, Poland and Norway. Upon his return, Stevin embarked on a successful public service career that was to include such positions as Surveyor of Taxes at Bruges, Quarter-master General of the Dutch army and Chief Inspector of Public Works. Stevin provides an excellent example of the Dutch practice of inducing individuals with the strongest intellectual abilities into careers in public service. Jan de Witt and Jan Hudde provide important 17th century examples of this practice.

Stevin is, perhaps, most legitimately remembered for his contributions to engineering mathematics, especially in the fields of hydrostatics and dynamics. He is also credited with inventing a practical canal lock and writing an important treatise on fortification. In theoretical mathematics, he is recognized for making substantive contributions to the development of symbolic algebra, such as being the first systematic user of decimal fractions and in extending this notion to fractional powers. In physics, Stevin produced a 1586 report, in Flemish, describing an experiment where two spheres of lead, one sphere ten times the weight of the other, were dropped from a height of 30 feet. Stevin's observation that the sounds of both the spheres striking the ground appear to be simultaneous predates a similar observation by Galileo, an observation that Galileo is often given credit for discovering.

Stevin had the personal misfortune of having to live during a period of hostilities between the Catholic Spanish crown and the Protestant Dutch Republic. In this dispute, Stevin took a public position squarely in line with the Protestant party of Mauritz, the Prince of Orange and, for a time, Stevin acted as tutor to the Prince. One of Stevin's

significant academic contributions was to write up the course of instruction that he gave to the Prince. The result was the *Hypomnemata Mathematica* (1605, 1608) ('Mathematical Traditions'), a work in two volumes with the Latin edition of the second volume being written by Stevin and published in 1605 and the first Latin volume being published in 1608 being a translation by Willebrond Snellius, a professor of mathematics at Leiden, from an original Dutch text by Stevin.

The text of the *Hypomnemata Mathematica* is constructed primarily to provide an overview of mathematics. However, in Part II of the second volume, Stevin produces *De Apologistica Principum Ratocino Italico* which roughly translates as 'Account-keeping for Princes after the Italian manner'. This treatise has been recognized as a key contribution to the history of double-entry bookkeeping, albeit not wholly successful in its application of double entry to public accounts. To quote Brown (1905, p.136):

> At the close of the sixteenth and the commencement of the 17th century there stands a figure that fitly represents the transition from the old order to the new in book-keeping. Simon Stevin ranks with Pacioli in this respect that he is a man of general learning who has not thought it beneath his dignity to compose a treatise on book-keeping.

As Holland was to be the 'training school' of many English merchants, the impact of the accounting standards established by Stevin extends far beyond the time and place of his writings.

Judging from the substance of his contributions, it is not difficult to recognize that Stevin was oriented to practical applications of mathematics. He saw little value in studies concerned with the more speculative and theoretical aspects of mathematics. This approach was not novel to Stevin but was representative of a school of thought associated with Petrus Ramus who advanced the view that much of what was taught in universities was overly complicated, too abstract and too theoretical. Intellectual study had to be focused on concrete, simple and practically oriented problems. For the study of mathematics, this meant that mathematical methods should focus on solving the problems of navigators, merchants, engineers and surveyors (van Berkel 1988, p.157). This was clearly the orientation that Stevin chose as the focus for his mathematical contributions.

Needless to say, Ramism was not popular among the humanist hierarchy that dominated the university system of Stevin's era. This hierarchy was comfortable with 'the social and intellectual barriers

between the theoretical science inside the university and the practical arts outside' (van Berkel 1988, p.158). Circa 1600, the humanist dominated university hierarchy typically judged mathematics with the contempt that was associated with the crafts. Yet, there was a pressing need for solutions to various mathematical problems that arose in military, engineering and commercial situations. This inherent social demand created a strong foundation for the spread of Ramism among progressive academics. An interesting connection between a major proponent of Ramism and the practical social need for mathematical training arises with the creation of the Dutch Mathematical School at Leiden in 1600. This connection was to play a key role in the history of financial economics in the mid-17th century.

The Dutch Mathematical School was initially designed as a school for military engineers. Founded by Mauritz, the Prince of Orange, it is not surprising that Simon Stevin would be chosen to design the curriculum and draw up the statutes for the new school. The school was to be situated in Leiden, affiliated with but not part of the University of Leiden. The affiliation was physical, with the school being housed in a university building. Members of the school were permitted access to the university library. Faced with the creation of the Dutch Mathematical school, the hierarchy at the University of Leiden were obliged to legitimize the role of practical mathematics in the university curriculum and to give adequate recognition to faculty teaching practical mathematics, such as the overt Ramist Rudolf Snellius. As such, the creation of the Dutch Mathematical School marks the beginning of a mathematical tradition that was soon to include Frans van Schooten (1615-1660) and his students Christian Huygens, Jan de Witt and Jan Hudde.

Commercial Arithmetic and the Ready Reckoner

The contributions of Simon Stevin mark a crossroads in the development of commercial arithmetic. By the early 17th century, the leading mathematical theorists had progressed markedly beyond the elementary concerns of commercial arithmetic. The era of Descartes, Fermat, Pascal, Leibnitz, Halley and Newton was about to begin. Reckoning masters were no longer cutting edge mathematicians. To be sure, with the emergence of probability theory in the late 17th century, mathematicians were drawn back to specific commercial problems, such as the pricing of aleatory contracts. However, after Stevin, those making substantive contributions in commercial arithmetic were no

longer those responsible for making pathbreaking contributions to mathematics. During the 17th century, progress in commercial arithmetic was largely in form, such as the adoption of decimal notation. By the end of the 17th century, the subject of commercial arithmetic was not much different than that in use today (D. Smith 1958, v.1, p.444).

As the extent and sophistication of mercantile activity increased, specialization of function emerged. By the late 16th and early 17th centuries, the major commercial centres appear to have had individuals specializing in compound interest calculations. In London, one such individual was Richard Witt, author of *Arithmeticall Questions* (1613). The full title of the text reveals its contents: 'Arithmeticall Questions, touching The Buying or Exchange of Annuities; Taking of Leases for Fines, or yearly Rent; Purchase of Fee-Simples; Dealing for present or future Possessions; and other Bargaines and Accounts, wherein allowance for disbursing or forebearance of money is intended; Briefly resolved, by means of certain Breviats.' The term 'breviats' is synonymous with 'tables'. The material contained in the book is sufficiently difficult that it would not have been practical to use as a text at a commercial academy. Hence, it would appear that Witt was an early 17th century version of an actuary, conveying his specialized knowledge to his peers.

Other than the reference on the title page to 'R.W. of London, practitioner in the Arte of Numbers', little is known about Richard Witt. The introduction to the second edition of *Arithmeticall Questions* (1634), produced by Thomas Fisher, provides the information that Richard Witt was dead in 1634 and that 'the Book almost forgot and out of use'. Murray (1930, pp.296-8) considers the book to be a **'ready reckoner'**, 'a book of completed calculations of various kinds, generally a Table of Products ... in use in shops and counting-houses, and commercial establishments of all kinds'. This characterization is unfair as the book does contain much more in the way of explanation and worked problems. Lewin (1970, p.130) is more judicious, claiming that Witt 'was probably the first to produce a practical text-book on the subject (of compound interest), with an extensive range of tables which readers could apply to their own problems'.

Murray's classification of the *Arithmeticall Questions* as a ready reckoner is also used to characterize other English works of that time that provided interest tables as part of a more extensive discussion of other matters, such as Thomas Clay's, *A Chronological discourse of the well ordering, disposing and governing of an honourable estate or*

revenue ... together with certain Briefe, easie and necessary Tables of Interest and Rents forborne; and also the Valuation of Leases (1618) with further editions in 1619 and 1624. Clay also produced *The Treasurers Almanacke, or the Money Master* (1624, 2nd ed.) which also contained interest tables. Throughout the 17th century, various other sources appeared in England and other countries which contained detailed compound or simple interest tables such as William Purser's *Compound Interest and Annuities* (1634), Johan Coutreels, *L'arithmetique de Jean Coutreels d'Anvers* (1626) and William Leybourn's *A Platform for Purchasers, a Guide for Builders, a Mate for Measurers* (1685).

William Leybourn (1626-1700), a printer who went on to become a teacher of mathematics, is credited with producing the original 'work which is the foundation of the Ready Reckoner of to-day; the author had a fancy for outlandish names and styled his book *Panarothmologia* and published it in London in 1693' (Murray 1930, p.300). Leybourn's book proved to be popular, in some cases even indispensable, and went into numerous editions, the last being in 1808. The term 'Ready Reckoner' is due to Daniel Fenning, an English schoolmaster, who published *The Ready Reckoner, or Traders' most useful Assistant* (1757), a book that went into numerous further editions. The term 'ready reckoner' appears in the title of numerous subsequent books. The title was so popular that it replaced the obscure *Panarothmologia* in the later editions of that book. Fenning produced numerous schoolbooks in his time, including *The Schoolmasters' most useful Companion* (1765) which has a section on bookkeeping.

Commercial Arithmetic, Accounting and Merchant Education

By the end of the 17th century, the theoretical development of commercial arithmetic was complete (D. Smith 1958, v.2). The subject, as captured in Renaissance texts such as the *Treviso*, had ceased to be of much interest to leading mathematicians even before this date. The *Treviso* was concerned with the calculations arising in merchant exchanges and in the merchant's counting house. Yet, by the late 17th century, mercantile calculations had become much more specialized than the calculations involved in the merchant's counting house. Commercial and financial applications, such as the life annuity valuation problem, contributed to mathematical developments outside the narrow subject matter of the 'commercial arithmetic'. To be sure, commercial arithmetic continued to play an essential role in the training of

merchants. Together with handwriting and accounting, commercial arithmetic made up the primary curriculum in the 18th century business academies (Murray 1930).

The intimate connection between handwriting, commercial arithmetic and accounting captured in the 18th century business academies extended a tradition stretching back to Pacioli and the chapter from the *Summa* on bookkeeping entitled, 'Of Reckoning and Writing'. This shared intellectual history of financial economics and accounting is a recurring theme. Some of the important contributors to financial economics, such as James Dodson, were also involved in merchant education, which required doing some instruction in accounting. Dodson, in particular, referred to himself with the appellation, 'Accomptant, and Teacher of Mathematics'. Important contributions to commercial arithmetic, proper, came increasingly from those involved in merchant education.

Along with the changes in commercial arithmetic, the educational process of training merchant apprentices also evolved. The traditions and curriculum of the Italian reckoning schools had to adapt to the types of arithmetical calculations and other operations needed in the merchant's counting house. As such, the educational mantle of the reckoning master gradually passed from the mathematician to the schoolmasters and accountants. Similarly, reckoning school education was increasingly the responsibility of schoolmasters and arithmetical textbook writers. By the early 17th century, financial economics was increasingly concerned with problems that differed from the subject matter of accounting. The intricate calculations of Richard Witt were well beyond all but specialized accountants.

During this period, accountants were struggling to gain practical acceptance for basic tools such as double entry (Yamey 1949, p.110):

> The evidence is largely against the view that the merchants of the period required anything more from their ledgers and journals than a clear and ready record of transactions for easy reference, and descriptive details of their cash, merchandise, and other assets bought and sold ... The majority of merchants were probably so intimately concerned with details of their own business affairs that they did not need elaborate accounting calculations to inform them of the size of their fortunes or to acquaint them of the results of their enterprise.

By the middle of the 18th century, the theoretical gap between accounting and financial economics was even wider. However, the emergence of publicly traded joint stock companies produced another type of connection:

> It is only when the firm is very large, or where the owner is not actively or continuously in control of his enterprise, that accounting data would be necessary to supply him, however imperfectly or inadequately, with information he would otherwise not possess.

Accurate analysis and control of the joint stock company requires accounting data. Hence, one of the most important valuation problems in financial economics is directly dependent on the interpretation of financial accounting information.

18th Century Theoretical Developments

The theoretical problems being solved during the 18th century were decidedly more sophisticated than those addressed in the commercial arithmetics of previous centuries. Detailed compound interest tables were now widely available in numerous sources, as was the knowledge to use such tables. Eighteenth century authors producing works containing traditional commercial arithmetic, both with and without compound interest tables, were typically schoolmasters or accountants, for example, Murray (1930). Theoretical progress in the 18th century was coming from solving the relatively new problems arising from the calculation of tables for valuing life annuities. The analysis required to address these problems was significantly more difficult than that used in, say, calculating compound interest tables.

Life annuity calculations needed accurate life tables, bringing demographics, statistics and, ultimately, actuarial science into the picture. After the establishment of the Equitable Life Assurance Society in 1762, the additional problem of calculating tables for life insurance premiums arose. Leading mathematical statisticians, such as Abraham de Moivre and the infamous Thomas Simpson (1710-1761), made important contributions to the solution of the various life annuity valuation problems. The cutting edge of financial economics had graduated from the relatively elementary concerns of commercial arithmetic to the rarefied atmosphere required to value aleatory contracts. The boundaries of financial economics now touched: accountancy, in which commercial arithmetic still played a primary role; mathematical statistics, demographics and probability theory, that were essential to the pricing of aleatory contracts; as well as the rapidly emerging subject of actuarial science.

The emergence of life annuity tables during the 18th century is reflected in the appearance of such tables in the works of de Moivre,

Richard Price, Thomas Simpson, Antoine Deparcieux (1705-1768) and others. The accuracy of life annuity tables depends fundamentally on the accuracy of the life table upon which the calculation of annuity values is based and a number of life tables, based on different populations, were presented during the 18th century. From the perspective of general demography, little progress was made in improving on the Breslau life table given by Edmond Halley in 1694. However, Deparcieux in 1746 and Willem Kersseboom (1691-1771) in 1738 did provide important information about the specific demographics of that segment of the population that had purchased life annuities.[2]

One of the curiosities of 18th century financial economics is John Smart the author of, arguably, the most important collection of compound interest tables produced during the 18th century, *Tables of Interest, Discount and Annuities* (1726). These tables, significant to nine figures, were likely copied and presented in simplified form to a lesser number of significant digits by de Moivre, Simpson and Deparcieux (Pearson 1978, p.203). In his *Tables*, Smart suggests an important improvement that could be made in order to improve the information in the London bills of mortality. In particular, Smart urged that Parish clerks give the age of Death. This suggestion was positively received and, making use of this additional information, Smart was also responsible for producing a life table from the London bills of mortality, based on the years 1728-1737. The original publication of this life table is uncertain, but it is fully reprinted in Corbyn Morris, *Observations of the Past Growth and Present State of the City of London* (1751).

What makes Smart's contributions such a curiosity? The short answer is Thomas Simpson. No other contributor to the history of statistics draws such venomous comments from Karl Pearson (1978, pp.166-7): 'That he was a man of some ability may be admitted, that he was wretchedly poor may be a partial excuse for his proceedings, but to me he is a distinctly unpleasant and truculent writer of cheap textbooks'. Though he has received favourable reviews from some later writers such as Augustus de Morgan (1847) and even Hald (1990), Simpson was probably one of the great plagiarizers of 18th century mathematics and statistics. Pearson is quite thorough in dissecting Simpson's contributions and assessing the actual sources for his writings. As Simpson is recognized for contributions in both mathematics and statistics, the number of accusers is considerable and the stories told makes for interesting reading, for example, Pearson (1978, p. 166-76).

In statistics, much of what Simpson produced was, apparently, taken from de Moivre. However, Simpson is credited with developing a life table. Richard Price in his *Observations* (1771, 1772), for example, has considerable discussion and some tables based on 'the probability of the duration of life in London, deduced by Mr. Simpson from observations on the bills of mortality in London for 10 years, from 1728 to 1737'. This raises the curious connection to John Smart, for it was Smart who developed the original life table from the improved data in the London bills of mortality. In *The Doctrine of Annuities and Reversions* (1742) Simpson observes that Smart's life table does not represent actual mortality because migration into and out of London has not been taken into account. In order to rectify this shortcoming, Simpson provides a 'hashed-up' (Pearson 1978, p.183) modification to Smart's table that even Hald (1990, p.518) says 'is so vague that it is impossible to see what he actually did'.

Appendix: Selected Compound Interest Tables

Compound interest tables are a reflection of the progress that had been attained in fixed income calculations. This appendix contains examples of compound interest tables from two sources: a breviat for *future value* using 10% interest taken from Witt (1613); and, the first page of a present value of annuity table from Price (1772) (Figures 4.3 and 4.4). The Witt breviat was made so early that the notation of placing a decimal or comma to denote a decimal fraction was not in use. Witt refers instead to treating the number in the breviat as a numerator, with the conversion to a decimal fraction being achieved by dividing by 10,000,000. The Price tables indicate the widening and deepening of interest calculations. In Price (1772), present value, not future value as in Witt (1617), is provided and the number of years has been increased from the 30 used by Witt to 100 years for single cash flows and 100 years, with inclusion of the perpetual value, for annuities.

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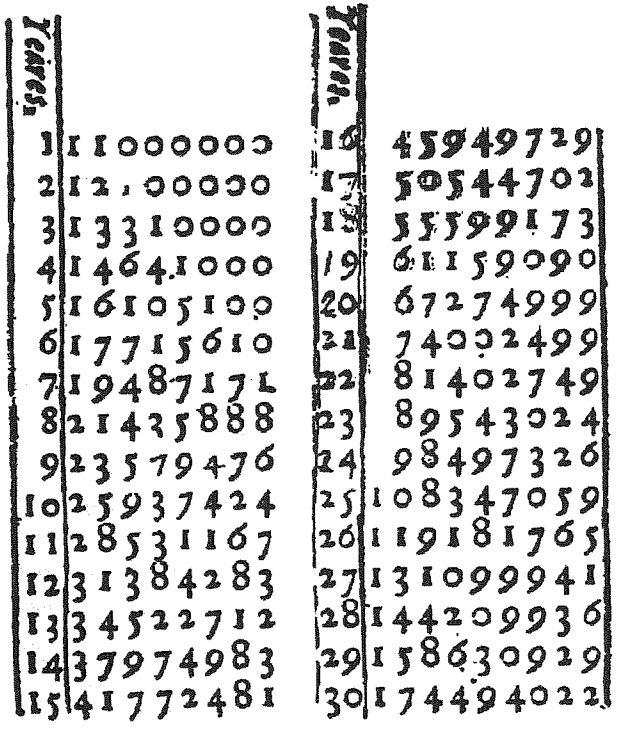
Figure 4.3 Breviat for compound interest calculations from Witt (1613)



The Breuiat of the Table of 10. per Cent.

Yeares.		Yeares.	
1	1 1 0 0 0 0 0 0	16	4 5 9 4 9 7 2 9
2	1 2 1 0 0 0 0 0	17	5 0 5 4 4 7 0 2
3	1 3 3 1 0 0 0 0	18	5 5 5 9 9 1 7 3
4	1 4 6 4 1 0 0 0	19	6 1 1 5 9 0 9 0
5	1 6 1 0 5 1 0 0	20	6 7 2 7 4 9 9 9
6	1 7 7 1 5 6 1 0	21	7 4 0 2 4 9 9
7	1 9 4 8 7 1 7 1	22	8 1 4 0 2 7 4 9
8	2 1 4 3 5 8 8 8	23	8 9 5 4 3 0 2 4
9	2 3 5 7 9 4 7 6	24	9 8 4 9 7 3 2 6
10	2 5 9 3 7 4 2 4	25	1 0 8 3 4 7 0 5 9
11	2 8 5 3 1 1 6 7	26	1 1 9 1 8 1 7 6 5
12	3 1 3 8 4 2 8 3	27	1 3 1 0 9 9 9 4 1
13	3 4 5 2 2 7 1 2	28	1 4 4 2 0 9 9 3 6
14	3 7 9 7 4 9 8 3	29	1 5 8 6 3 0 9 2 9
15	4 1 7 7 2 4 8 1	30	1 7 4 4 9 4 0 2 2

Theſe Numbers in this Breuiat muſt alſo be eſteemed Numerators, each of them hauing for Denominator 1 0 0 0 0 0 0 0.

Source: This image was adapted from Lewin (1970).

Figure 4.4 Compound interest calculations from Price (1772)

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TABLE II.

The preſent Value of an Annuity of One Pound, for any Number of Years not exceeding 100, at the ſeveral Rates of 3, 3½, 4, 5, and 6 *l. per Cent.*

Ye.	3 per Ct.	3½ per Ct.	4 per Ct.	5 per Ct.	6 per Ct.
1	.9708	.9662	.9615	.9523	.9433
2	1.9133	1.8997	1.8860	1.8594	1.8333
3	2.8286	2.8016	2.7750	2.7232	2.6730
4	3.7170	3.6731	3.6298	3.5459	3.4651
5	4.5797	4.5151	4.4518	4.3294	4.2123
6	5.4971	5.3286	5.2421	5.0756	4.9173
7	6.2302	6.1145	6.0020	5.7863	5.5823
8	7.0196	6.8740	6.7327	6.4632	6.2097
9	7.7861	7.6077	7.4353	7.1078	6.8016
10	8.5302	8.3166	8.1108	7.7212	7.3600
11	9.2526	9.0015	8.7604	8.3064	7.8868
12	9.9540	9.6633	9.3850	8.8632	8.3838
13	10.6349	10.3027	9.9856	9.3935	8.8526
14	11.2960	10.9205	10.5631	9.8986	9.2949
15	11.9379	11.5174	11.1183	10.3796	9.7122
16	12.5611	12.0941	11.6522	10.8377	10.1058
17	13.1661	12.6513	12.1656	11.2740	10.4772
18	13.7535	13.1897	12.6592	11.6895	10.8276
19	14.3238	13.7098	13.1339	12.0853	11.1581
20	14.8774	14.2124	13.5903	12.4622	11.4699
21	15.4150	14.6980	14.0291	12.8211	11.7640
22	15.9389	15.1671	14.4511	13.1630	12.0415
23	16.4436	15.6204	14.8568	13.4885	12.3033
24	16.9355	16.0584	15.2469	13.7986	12.5503
25	17.4131	16.4815	15.6220	14.0939	12.7833

Note: Price (1772) is an excellent example of the state of compound interest calculations around the time that the *Wealth of Nations* was published. The contrast with Witt is striking.

Notes

1. Another significant feature of the *Treviso* was that the book was published in the Venetian republic and not in Germany, the region where printing originated around 1450 with Gutenberg and Fust.

2. Restricting the population used in calculating a life table is an essential element in pricing an insurance premium. This follows because it is not the life table for the whole population which is needed in pricing insurance (or life annuities) but, rather, that segment of the population which purchases insurance.